applications than upon the logical rigor with which fundamental principles are established. The book of Miller and Lilly seems to be decidedly usable as a class textbook, and is likely to find favor among teachers.

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PROBLEMS AND SOLUTIONS.

Edited by B. F. Finkel and R. P. Baker.

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PROBLEMS FOR SOLUTION.

ALGEBRA.

468. Proposed by H. C. FEEMSTER, York College, Nebraska.

In each of the following series find the nth term and sum:

(a) $$2 + 5 + 9 + 15 + 24 + \cdots$$
(b) $$1 + 6 + 10 + 20 + 35 + \cdots$$
(c) $$1 + 5 + 15 + 35 + 70 + \cdots$$

469. Proposed by T. H. GRONWALL, New York City.

Show that the equation

$$f(x) = 2ax^4 + (1 - b)x^3 + b(1 - b)x - 2ab = 0,$$

where $0 < b < 1$, $a > 0$ and $a^2 > b$ has only one positive root and that this root lies between the roots of $g(x) = x^2 - 2ax + b = 0$.

470. Proposed by ERNEST W. BROWN, Yale University.

There are n numbers each lying between $-\frac{1}{2}$ and $+\frac{1}{2}$, such that any value of each between these limits is equally probable. What is the probability that their sum will lie between $s - \frac{1}{2}$ and $s + \frac{1}{2}$, where s is an integral multiple of $\frac{1}{2}$?

GEOMETRY.

499. Proposed by NATHAN ALTSHILLER, University of Oklahoma.

Find the surfaces all the plane sections of which are circles.

500. Proposed by R. T. McGREGOR, Bangor, California.

$OABC$, $OA'B'C'$ are two straight lines such that AA', BB', CC' are parallel. AB', $A'C$ meet in P; $A'B$ and AC' meet in Q. Show by synthetic projective geometry that PQ is parallel to AA'. Milne's *Projective Geometry*, Chap. I, Ex. 20.

501. Proposed by R. P. BAKER, University of Iowa.

Find the minimum amount of lumber one inch thick required to pack a gross of spheres three inches in diameter in a rectangular box.

502. Proposed by R. P. BAKER, University of Iowa.

A designer of machinery requires a curve having the following properties:

(1) A closed curve touching a given circle at two diametral points and enclosing it.
(2) The sum of the three radii from the center of this circle to the curve which make with each other angles of 120° is constant.

(3) The locus of a point which lies at some constant distance from the curve on its inner normal must be such that it is also the locus of a point fixed on a bar of some simple linkage. In estimating the value of the word "simple" pivoted bars are preferred to slides and the total number should be as small as possible.

Condition (3) is needed to enable a cylinder to be ground accurately to the curve.

CALCULUS.

417. Proposed by H. S. UHLER, Yale University.

To the degree of approximation indicated, show that

$$(\sqrt{-1})^{\sqrt{-1}} = 0.207,879,576,351.$$

418. Proposed by B. F. FINKEL, Drury College.

A rectangular tract of land is to be bought for the purpose of laying out a quarter-mile track with straightaway sides and semicircular ends. In addition a strip 35 yards wide along each straightaway is to be bought for grandstands, training quarters, etc. If the land costs \$200 an acre, what will be the least possible cost of the land required?

GRANVILLE'S *Differential and Integral Calculus*, p. 116.

Is there anything wrong with this problem? Explain the contradiction involved in the solution.

MECHANICS.

334. Proposed by HORACE OLSON, Chicago, Illinois.

A particle of elasticity e is projected with a velocity v at an angle ϕ with a plane inclined to the horizontal at an angle ψ; its plane of motion is perpendicular to the inclined plane. Show that after $2v \sin \phi/g(1 - e) \cos \psi$ seconds it will cease to rebound and will move along the plane with an initial velocity $v \cos \phi - 2v \sin \phi \tan \psi/(1 - e)$ and a uniform acceleration, $g \sin \psi$, down the plane.

(This problem is a generalization of problem 289 in Mechanics, which appeared in the March, 1914, issue of the MONTHLY.)

335. Proposed by HAROLD T. DAVIS, Colorado Springs, Colorado.

A heavy particle is projected upwards with a velocity V in a medium resisting as the nth power of the velocity. Prove that the elevation of the particle when the velocity downwards is V is equal to LT where L is the limiting velocity and T is the time in which the particle falling from rest in the medium will acquire a velocity V^2/L.

NUMBER THEORY.

254. Proposed by HORACE OLSON, Chicago, Illinois.

Find three integers, x, y, and z, such that $x^2 + y^2$, $x^2 + z^2$, $y^2 + z^2$, and $x^2 + y^2 + z^2$ are all perfect squares.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

456. Proposed by PAUL CAPRON, U. S. Naval Academy.

If

$$S_{i,n} = \sum_{k=1}^{k=n-i+1} \frac{(i + k - 1)!}{(k - 1)!},$$

show that $S_{i,n}$ is equal to $1/(i + 1)$ times the last term of $S_{i+1,n+1}$; as, for instance, that

$$S_{1,n} = 1 + 2 + \cdots + n = \frac{n}{2}(n + 1),$$

that

$$S_{2,n} = 1\cdot 2 + 2\cdot 3 + \cdots + n(n + 1) = \tfrac{1}{3}(n - 1)n(n + 1),$$

etc.